BRL-TR-3064

(4)

TECHNICAL REPORT BRL-TR-3064

BRL

AD-A215 445

POSSIBILISTIC VULNERABILITY MEASURES

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DECEMBER 1989

U.S. ARMY LABORATORY COMMAND

BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for Public Release; Distribution Unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) BRL-TR- 3064	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION USA Ballistic Research Laboratory 6b. OFFICE SYMBOL (If applicable) SLCBR-VL-G	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code) Aberdeen Proving Ground, MD 21005-5066	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)

POSSIBILISTIC VULNERABILITY MEASURES

12. PERSONAL AUTHOR(S)

Aivars Celmiņš

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
FINAL	FROM ______ TO ______		

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Measures of Vulnerability; Vague Data; Degree of Possibility; Possibility Theory; Degree of Necessity; Possibilistic Pattern Matching; Possibilistic Kill Criterion

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Possibility theory has been developed to handle data that are vague but not necessarily probabilistic. Because this type of information is characteristic for vulnerability problems, possibility theory is particularly well suited for applications in vulnerability analysis. This report provides a short introduction to the theory and describes how possibilistic concepts can be used to analyze vulnerabilities. The essential result is the establishment of measures for the possibility and necessity of kill of a system in presence of a threat. The system, the threat and the responses of elementary system components are approximately defined in terms of fuzzy sets. This information is used to specify a possibilistic pattern matching problem the solution of which are degrees of possibility and necessity that the system is killed. The advantage of these measures of vulnerability is that they indicate not only whether a system is intact or damaged, but also how close the system is to the threshold of damage.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL Aivars Celmiņš	22b. TELEPHONE (Include Area Code) 301-278-6986 22c. OFFICE SYMBOL SLCBR-VL-G

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1. Introduction

A theory of possibility was first proposed by Zadeh in 1978 (Reference 1) as a means to quantify judgement. The theory provides a formalism to treat vague, imprecise or incomplete information which is different from the approaches by probability theory and interval analysis. Each of these three theories emphasize different aspects of vagueness and thus complement each other. Possibility theory is particularly adequate for the development of vulnerability measures because, first, the basic information in vulnerability problems is typically vague but not random and, second, the theory provides a natural means to rank the relative outcomes from a combination of targets and threats. A comprehensive summary of the present status of possibility theory is given in Reference 2.

The relation between possibility theory and interval analysis is easy to describe because interval analysis is a limit case of possibility theory. More precisely, for interval analysis one assumes that an event either does or does not belong to a certain category, whereas in possibility theory the event is allowed to belong more or less to the category. This is expressed by assigning to the event and to its contrary a lesser or larger degree of possibility (between 0 and 1). These two possibilities are only weakly related: if an event is possible, then this does not prevent its contrary to be possible, too. (The law of the excluded middle is weakened in the context of possibility measures). The complement (with respect to 1) of the degree of possibility of the contrary event can be interpreted as the necessity (certainty) of the event itself. Thus, in possibility theory an event is characterized by two possibility measures — a degree of possibility Π and a degree of necessity N.

The relation between possibility theory and probability theory is more esoteric. They are connected via the concepts of plausibility and belief within the framework of Dempster-Shafer theory. In order to handle incomplete data Dempster introduced the concepts of lower and upper probabilities. The actual probability of an event (with complete and disjoint elements) falls between these limits. If the data are crisp and certain other requirements are satisfied, then the lower probability (belief) corresponds to a degree of necessity and the upper probability (plausibility) corresponds to a degree of possibility. Hence, if it makes sense to assign a probability to an event, then it likely has a value somewhere between the degrees of necessity and possibility. The generalization which is achieved by using possibilistic instead of probabilistic measures provides a natural way to handle data which are not exact, but given, e.g. by an interval. Probability measures generally apply to precise but differentiated items, whereas possibility measures are more adequate for the representation of imprecise and possibly coherent information. The most striking difference between probability and possibility is that the probability $P(A)$ of an event completely determines the probability $P(\bar{A})$ of its contrary: $P(\bar{A}) = 1 - P(A)$. The corresponding possibility measures satisfy only the relations $\Pi(A) + \Pi(\bar{A}) \geq 1$ and $\mathrm{N}(A) + \mathrm{N}(\bar{A}) \leq 1$.

In applications to vulnerability analysis one is particularly interested in possibilistic pattern matching. Indeed, in order to determine whether a system component $e \in E$ is killed in the presence of a threat $t \in T$, one might define in the event space $G = (E \times T)$ a set K of all such events for which the system component is assumed to be killed, and then check whether the given element $g = (e, t)$ belongs to K. If the descriptions of g and K are vague then g might match an element of K to a larger or lesser degree. Possibility theory allows one to associate with such a partial matching a degree of possibility $\Pi_K(g)$ and a degree of necessity $\mathrm{N}_K(g)$ that e is killed. Once Π_K and N_K are known for all components of a system, then the corresponding Π_K and N_K for the whole system are computed by aggregation of the individual results using logical "and/or" rules in a fault tree.

This report is a short description of the concepts and techniques that are needed for the application of possibility theory to vulnerability problems. In Section 2 we discuss the computation of kill possibility measures for an elementary system component. Section 3 outlines the aggregation of elementary possibilities to obtain the possibility measures for a kill of a system.

2. Possibilistic Vulnerability Criterions.

2.1. Crisp Kill Criterions.

To determine the vulnerability of a system, generally the system is broken down into components, which are in turn broken down into subcomponents, etc. until at the lowest level one has a set of *elementary components*. The state of each elementary component is assumed to be either "intact" or "killed", and is indicated by a 1 or 0, respectively, in a *state vector* where each vector component represents the state of an elementary system component. The state of the system is calculated from the binary state vector of the elementary components by logical aggregation which successively yields state indicators for higher level system components. At those levels, the state indicators generally can have values in the interval [0,1] indicating the level of operability of each system component. (Alternatively the computations might yield probabilities of damage. We shall not discuss the pro and contra of various methods of aggregation and/or interpretation of the results. Our interest is in the general approach which consists of an estimation of the state of low level components and aggregation to obtain the state of higher level components). The final result of the aggregation algebra are indicators of the state of the whole system. If the system encounters a specific threat then the state vector of the elementary components changes and the aggregation yields state indicators of the new state of the system due to the threat. Hence one basic problem in vulnerability estimates is the definition of a threshold for the threat above which the state of an elementary component is affected.

Let the kill criterion of an elementary component be expressed by the equation

$$C(e,t) \geq 0 \quad , \tag{2.1}$$

where e is a parameter describing the component and t is a parameter which specifies the threat. $C(e,t)$ is a decision function (in a probabilistic model it may also depend on random variables). Let it determine the outcome of a pairing of e and t by the convention

$$\begin{aligned} C(e,t) < 0 &\rightarrow \text{component intact} \\ C(e,t) \geq 0 &\rightarrow \text{component killed} \end{aligned} \tag{2.2}$$

or

$$S^{(1)}(e,t) = \begin{cases} S^{(0)}(e) & \text{if} \quad C(e,t) < 0 \ , \\ 0 & \text{if} \quad C(e,t) \geq 0 \ , \end{cases} \tag{2.3}$$

where $S^{(0)}(e)$ and $S^{(1)}(e,t)$ are the states of the elementary component before and after application of the threat, respectively.

We illustrate in Figure 1 the notion of a decision function in a case where e and t are scalar parameters, and the zero level of the decision function defines a curve in the e,t-plane. A specific combination of component and threat is given by particular values for e and t which define a point in the event plane. Depending on the location of that point with respect to the decision curve $C = 0$ the component is assumed to be either intact or killed.

Several generalizations of this situation are needed in practice aside from the obvious that e and t usually are vectors. First, the descriptors e and t might be known only approximately and, second, the decision function C might be only defined within some bounds. We assume that these inaccuracies are defined in terms of fuzzy numbers and functions, that is, by membership functions which express the possibility of the occurrence of a specific threat t and target element e, and which define possible locations of the decision bound $C(e,t)=0$ in the event space.

As an example, assume that the caliber of a threat projectile is known to be within certain bounds. Then we assign positive membership values (less or equal 1) to calibers within these bounds and zero membership values to all other calibers. These membership values then define the fuzzy set "caliber of the threat". Approximate target descriptions are handled correspondingly. Approximate decision functions define, instead of a crisp decision surface as a solution of the equation $C(e,t)=0$, a fuzzy surface as the solution of a fuzzy equation which represents a gradual transition between "kill" and "intact" situations. We assume that the gradual transition again is expressed by a corresponding membership function. An example of such a representation is given in the next section.

2.2. Possibilistic Decision Bounds.

One can treat the kill criterion (2.2) as a pattern matching problem because the condition $C(e,t)\geq 0$ defines in the event space $G=(E\times T)$ a subset K of unfavorable combinations of e and t. Given a particular event $g=(e,t)$ one is interested to determine whether this datum matches any of the elements in K, that is, whether $g\in K$. This is a pattern matching problem. If g and/or K are only approximately given, then a matching to a larger or lesser degree is a natural outcome. Measures of the degree of matching are provided by the concepts of possibility theory (Reference 3). In this section we consider the special case where the decision function is fuzzy and the events g are crisp. In the next section we shall consider the general case where the decision function as well as the events are fuzzy.

We indicate fuzzy functions by a tilde and let the decision be specified by a fuzzy function $\tilde{C}(e,t)$. The decision equation $\tilde{C}(e,t)=0$ defines in the event space a *fuzzy surface* (*Reference* 4) which is the fuzzy bound of the set $\tilde{K}$ containing the "kill" patterns. (The set is fuzzy because its boundary is fuzzy). Figure 2 shows a fuzzy bound in a one-dimensional event space, where $\tilde{C}=0$ defines a fuzzy number, or a "fuzzy point" as a fuzzy segment of the number line. For convenience we shall assume in our examples that fuzzy numbers have trapezoidal membership functions as shown in the figure. The interval $[x_1,x_2]$ is called the *core* of the number, and the interval $[x_1-x_3\ ,\ x_2+x_4]$ is called the *support* of the number. Fuzzy numbers $\tilde{x}$ with this type of membership functions can be conveniently represented by the quadruple (x_1,x_2,x_3,x_4). If the number is crisp, say x_0, then its representation is $(x_0,x_0,0,0)$. In the present example the fuzzy number in Figure 2 is meant to be the solution of a fuzzy equation $\tilde{C}(x)=0$. We assume that there exists a corresponding crisp equation $C(x)=0$ which has a solution x_0 within the core of the fuzzy solution.

Let $\mu_y(x)$ be the membership function of a fuzzy number $\tilde{y}$. The *degree of possibility* that a crisp number a is smaller than $\tilde{y}$ (is to the left of $\tilde{y}$) is a fuzzy number with the membership function (Reference 2)

$$\rho_{a \le \tilde{y}}(a) = \sup_{\xi \le a} \mu_{\tilde{y}}(\xi) \ . \tag{2.4}$$

The *degree of necessity* that a is smaller than $\tilde{y}$ is defined by the membership function

$$\nu_{a \le \tilde{y}}(a) = \inf_{\xi \ge a} \left(1 - \mu_{\tilde{y}}(\xi) \right) \ . \tag{2.5}$$

Figure 3 shows these possibility and necessity membership functions for a fuzzy bound $\tilde{y}$ with the membership function shown in Figure 2. If the bound y is a crisp number then there is only one crisp segment of the number line where the crisp numbers are less than y. In contrast, if the bound is fuzzy then there are two fuzzy segments, defined by the membership functions ρ and ν, and representing the possibility and necessity, respectively, that a crisp number is less than $\tilde{y}$.

The generalization to a multi-dimensional situation is simple. Let the bound of K be given by a membership function $\mu_{\tilde{C}=0}(e,t)$, and let the corresponding crisp equation $C(e,t)=0$ define a crisp decision bound within the core of $\mu_{\tilde{C}=0}$. Then the possibility that the pairing $g=(e,t)$ results in a kill of the component e is

$$\rho_{\tilde{K}}(e,t) = \rho_{g \in \tilde{K}}(e,t) = \begin{cases} \mu_{\tilde{C}=0}(e,t) & \text{if } C(e,t) < 0 \ , \\ 1 & \text{if } C(e,t) \ge 0 \ . \end{cases} \tag{2.6}$$

The necessity that the pairing g results in a kill of e is

$$\nu_K(e,t) = \nu_{g \in \tilde{K}}(e,t) = \begin{cases} 0 & \text{if } C(e,t) < 0 \ , \\ 1 - \mu_{\tilde{C}=0}(e,t) & \text{if } C(e,t) \ge 0 \ . \end{cases} \tag{2.7}$$

The membership functions ρ_K and ν_K define in the event space the fuzzy subsets of possibility and necessity, respectively, that a crisp event corresponds to a kill. We call these subsets $\tilde{K}_\Pi$ and $\tilde{K}_N$. If the decision criterion is crisp then the subsets are identical ordinary crisp subsets.

2.3. Possibilistic Events.

We now consider the case where the component as well as the threat are given by sets of possible values of e and t. Let the functions $\pi_E(e)$ and $\pi_T(t)$ be the degrees of possibility that e and t actually occur. Given those possibilities, one would like to know the possibility and necessity that the event $g=(e,t)$ corresponds to a kill situation.

In the event space G the possibility of the event $\tilde{g}=(\tilde{e},\tilde{t})$ has the membership function

$$\pi_G(e,t) = \min \{ \pi_E(e), \pi_T(t) \} \ . \tag{2.8}$$

If e and t are scalars, and $\pi_E(e)$ and $\pi_T(t)$ are trapezoidal functions then $\pi_G(e,t)$ is a frustrated pyramid in the e,t,π-space as shown in Figure 4. $\pi_G(e,t)$ defines a fuzzy subset $\tilde{g}$ in the event space. In Figure 4 this subset is a fuzzy point in a two-dimensional space. The possibility that this point corresponds to a kill situation is measured by the degree of possibility that $\tilde{g}$ matches a point within the subset $\tilde{K}_\Pi$ defined by $\tilde{C}(e,t)>0$. That possibility is calculated as follows (Ref. 2, p.25; Ref. 3, p.316):

$$\Pi_K(\tilde{g}) = \Pi_{\tilde{g}\subseteq\tilde{K}} = \sup_{e,t} \; \min \{ \pi_G(e,t), \rho_K(e,t) \} \;, \qquad (2.9)$$

where ρ_K is the membership function of the subset $\tilde{K}_\Pi$. The necessity of a match is calculated by using the membership function ν_K of the subset $\tilde{K}_N$ and the formula

$$N_K(\tilde{g}) = N_{\tilde{g}\subseteq\tilde{K}} = 1 - \sup_{e,t} \; \min \{ \pi_G(e,t), 1-\nu_K(e,t) \} \;. \qquad (2.10)$$

Figure 5 shows a number of possible situations in the case where the event space is two-dimensional and the decision function $C(e,t)$ is crisp. The event is given by a membership pyramid similar to that in Figure 4. A number of different locations of the decision curve are shown as dashed lines. The degrees of possibility and necessity that the event belongs to the subset $C\geq 0$ depends on the relative locations of event and decision curve, and the decision curves are labeled correspondingly. The figure shows how to interpret the possibility and necessity measures. The possibility is positive if a part of the support of $\tilde{g}$ is inside K, and it equals 1 if at least a part of the core is inside K. The necessity is zero unless the whole core is inside K. It equals 1 only if the whole support of $\tilde{g}$ is inside K.

Trapezoidal membership functions are well suited for the discussion and illustration of possibilistic concepts. However, a restriction to trapezoidal fuzzy numbers has also practical numerical advantages because with this restriction and a decision function which yields the values of $\rho_K(e,t)$ and $\nu_K(e,t)$ for any set (e,t) one can compute Π_K and N_K by fast search algorithms.

2.4. Example.

We present an example of combined kill criterions for an armor plate subjected to blast. For this example, we consider two types of damage: a failure of the plate and a deflection greater than a threshold. The plate (or a component behind the plate) is "killed" if either of these criterions is satisfied.

Both criterions are formulated in terms of the following dimensionless variables.

$$\left.\begin{aligned} A &= \frac{z}{s} \ , \\ B &= \frac{E}{s^2 h \sigma_y} \ , \end{aligned}\right\} \tag{2.11}$$

where z = deflection of the plate after loading,
s = distance of the charge from the plate,
E = energy of the charge,
h = thickness of the plate,
σ_y = yield stress of the plate's material.

Let the failure bound be given by the equation (Reference 5)

$$C_1 = B - B_0 = 0 \ , \tag{2.12}$$

let the deflection be given by the function

$$\frac{z}{s} = a + b\, B^{1.5} \ , \tag{2.13}$$

and let the damage criterion due to a large deflection be given by the equation

$$C_2 = a + b\, B^{1.5} - A = 0 \ , \tag{2.14}$$

where $A = z_{crit}/s$ is a critical value for the deflection.

In the event space, the component (the plate) is represented by the vector

$$e = (\, h\sigma_y \ , z_{crit} \,) \tag{2.15}$$

and the threat is defined by the vector

$$t = (\, E, s \,) \ . \tag{2.16}$$

Hence, the event space is four-dimensional in this example. The subset of killed components in the event space is defined by

$$(e,t) \in K \qquad \text{if} \qquad C_1(e,t) \geq 0 \ , \quad \text{or} \quad C_2(e,t) \geq 0 \ . \tag{2.17}$$

The analysis can be simplified by mapping the four-dimensional event space to a two-dimensional space with the dimensionless coordinates A and B. The mapping is provided by eq. (2.11). The parameters a, b and the exponent 1.5 in eq. (2.14), and the parameter B_0 in eq. (2.12) are fuzzy numbers. Therefore the decision bound defined by eq. (2.17) is a fuzzy curve in the B,A-plane. It is shown in Figure 6. The support of the decision bound is the area between the two outermost curves. The core of the bound is the curve running approximately in the center between the support boundaries. It corresponds to the line $\mu_{C=0} = 1$ (the crisp solution of the decision equations). The straight vertical lines in the top part of the figure represent the failure bound (2.12) and the curved part of the bound represents eq. (2.14). Events corresponding to a kill of the component are in the lower right part of the figure. In that area the membership values of the possibility of kill, ρ_K, and necessity of kill, ν_K, both equal 1. Moving to the left, the necessity membership ν_K reduces to zero at the center curve, at which point ρ_K starts to decline and reaches zero at the leftmost curve. The two surfaces $\rho_K(B,A)$ and $\nu_K(B,A)$ define for a crisp event (B,A) the degrees of possibility and necessity of kill.

In real applications, the input values of A and B are fuzzy. For instance, when one computes the vulnerability of a given vehicle, then one might have the exact values of the parameters $h\sigma_y$ and z_{crit}, but only a possibility membership function for the parameters E and s of the threat. Then both coordinates A and B of the event are fuzzy because both depend on the fuzzy threat parameters. Moreover, because A as well as B depend on the same parameter s of the threat they are "interactive" in the terminology of fuzzy sets. Figure 7 shows three examples of such fuzzy events. We have assumed in all three cases the same fuzzy value $\tilde{E}$ of the energy of the charge and varied its distance $\tilde{s}$ from case to case by the (crisp) factor 1.32. Because the two coordinates $\tilde{B}$ and $\tilde{A}$ of the events are interactive, the supports and cores of the events resemble slanted parallelograms instead of being rectangles as in Figure 4. The boundaries of the decision bound from Figure 6 are plotted in Figure 7 as dashed curves. One observes that the degrees of possibility and necessity of kill increase as the distance between the charge and target decreases, that is, as the event moves to the right and into the subset corresponding to $C>0$.

3. Aggregation of Possibilistic Measures.

Section 2 describes the computation of the degrees of possibility and necessity that an elementary system component is killed. The corresponding indicators for higher level components and for the whole system are computed by aggregation operations on the vectors of state, possibility and necessity of the elementary system components. Now we shall discuss such aggregations.

The initial state of an elementary system component $e \in E$ is given by its state value $S^{(0)}(e)$ which is either 0 or 1. The threat defines an event space $G=(E \times T)$, and the decision criterions for the components e define in the event space subsets K where a kill of the elementary system components is possible. Actually, because we compute the possibility as well as the necessity of a kill, and assume a fuzzy decision criterion we have two such subsets which we call $\tilde{K}_\Pi$ and $\tilde{K}_N$, respectively. (If the criterion is crisp, then the two subsets are crisp and identical). The possibility that a fuzzy event $\tilde{g}$ corresponds to a kill of $\tilde{e}$ is in terms of the initial state $S^{(0)}$ of the component and the possibility Π_K that $\tilde{g} \in \tilde{K}_\Pi$ given by the formula

$$\Pi_K^{(1)}(\tilde{g}) = 1 - S^{(0)}(\tilde{e})\cdot(\, 1 - \Pi_K(\tilde{g})\,) \quad . \tag{3.1}$$

The *necessity that the event corresponds to a kill of* $\tilde{e}$ is

$$N_K^{(1)}(\tilde{g}) = 1 - S^{(0)}(\tilde{e})\cdot(\, 1 - N_K(\tilde{g})\,) \quad , \tag{3.2}$$

where N_K is the degree of necessity that $\tilde{g} \in \tilde{K}_N$. Hence, after the application of a threat (pairing the initial state of the system with a threat), the state of each elementary system component is characterized by two numbers $\Pi_K^{(1)}$ and $N_K^{(1)}$ in the interval [0,1] instead of the one binary state indicator $S^{(0)}$. If everything is crisp then eqs. (3.1) and (3.2) reduce to eq. (2.3) and the new state of the system is indicated by the binary state vector $S^{(1)}$. The possibilistic indicators provide additional information, in essence telling one how close the event is to a threshold of damage.

The degrees of possibility and necessity of killing a higher level component f are calculated in two steps. First, the results at the elementary level are aggregated to obtain transfer values Π_K and N_K for the higher level component. Then, these quantities are used in equations (3.1) and (3.2) to calculate the possibility measures for f. The transfer values are obtained as follows.

Let $q=(f,t)$ be the event in terms of the component and threat combination for a higher level component with the parameter f. For simplicity we omit the tilde which indicates fuzziness and assume from now on that all entities can be either crisp or fuzzy. The aggregation of the transfer functions depends on the type of dependency of f on the elementary components e. The simplest rules of aggregation are those for a pure conjunction and pure disjunction. If the aggregation of elementary system components to the next level is a conjunction (component f is killed if all components e_1 through e_m are killed) then the transfer values of the possibility and the necessity are aggregated by min operations (Ref. 3, p. 318):

$$\Pi_K(q) = \min_{i=1,\ldots,m} \Pi_K^{(1)}(g_i) \quad , \tag{3.3}$$

$$N_K(q) = \min_{i=1,\ldots,m} N_K^{(1)}(g_i) \quad . \tag{3.4}$$

If the aggregation is disjunctive (f is killed if either of the components e_1 through e_r is killed), then the max operator is used:

$$\Pi_K(q) = \max_{i=1,\ldots,r} \Pi_K^{(1)}(g_i) \quad , \tag{3.5}$$

$$N_K(q) = \max_{i=1,\ldots,r} N_K^{(1)}(g_i) \quad . \tag{3.6}$$

In practical applications, one needs more flexibility than provided by these standard aggregations. Such a flexibility is obtained by attaching weights to the lower level results, whereby the weights express the relative importance of a kill of elementary level components for the kill of a higher level component.

Let e_i, $i=1,\ldots,n$ be the set of all elementary system components and g_i be the corresponding elements of the event space. Let $w_i \in [0,1]$ be weights expressing the relative importance of the decision function C_i, that is of the state of the element e_i, for the aggregation with respect to the higher level component f. (A larger weight means that the killing of the particular component is more important; in terms of pattern matching, the matching of a component with a larger weight is more significant). Let the weights be normalized by

$$\max_{i=1,\ldots,n} w_i = 1 \quad . \tag{3.7}$$

The transfer values are obtained in case of weighted conjunction by the minimization (Ref. 3, p.322)

$$\Pi_K(q) = \min_{i=1,\ldots,n} \max \{ 1-w_i \ , \Pi_K^{(1)}(g_i) \} \quad , \tag{3.8}$$

$$N_K(q) = \min_{i=1,\ldots,n} \max \{ 1-w_i \ , N_K^{(1)}(g_i) \} \quad . \tag{3.9}$$

The corresponding aggregation for the weighted disjunction is

$$\Pi_K(q) = \max_{i=1,\ldots,n} \min \{ w_i \ , \Pi_K^{(1)}(g_i) \} \quad , \tag{3.10}$$

$$N_K(q) = \max_{i=1,\ldots,n} \min \{ w_i \ , N_K^{(1)}(g_i) \} \quad . \tag{3.11}$$

Eqs. (3.7) through (3.11) contain the unweighted aggregations as a special case with $w_i \equiv 1$.

The degrees of possibility and necessity that f is killed are computed from the transfer values in analogy to eqs. (3.1) and (3.2) by

$$\Pi_K^{(1)}(q) = 1 - S^{(0)}(f) \cdot (1 - \Pi_K(q)) \tag{3.12}$$

and

$$N_K^{(1)}(q) = 1 - S^{(0)}(f)\cdot(1 - N_K(q)) \ , \tag{3.13}$$

where $S^{(0)}(f)$ is the initial state of the system component f.

The weighted aggregation is a convenient method to compute different kill categories from the same damage state of the elementary components. Thus, the contribution of a component e_i or subsystem f_i to a firepower kill often will differ from its contribution to a mobility kill, or to a specific mission kill. The weighted aggregation provides more flexibility than a categorical assignment of the components to groups corresponding either to "contribution" or to "no contribution" to a specific type of kill.

4. Conclusions.

In vulnerability analysis, the available information often is imprecise but not random, for instance if the threat or the target is a future system or when one seeks to estimate the vulnerability of a system to threats within a finite range. The analysis of this type of data is the subject of possibility theory which defines measures for the possibility and the necessity that an approximately described system is killed in the presence of an approximately described threat. Of these two possibilistic measures, the degree of necessity is more important for applications in vulnerability, because a positive necessity measure means that one can be more or less certain that a kill takes place.

The input for vulnerability estimates based on possibility theory consists of approximately given threats and approximately described systems in addition to approximate decision functions for the damage of elementary components of the system. The user has a great freedom in defining the uncertainties and the theory takes into account all these inaccuracies in a semantically correct manner. Equally flexible is the aggregation of possibilistic damage indicators of elementary system components to obtain the degrees of possibility and necessity that the whole system is killed. By using proper weights in the aggregation process one can obtain the results for various kill categories with the same computer program.

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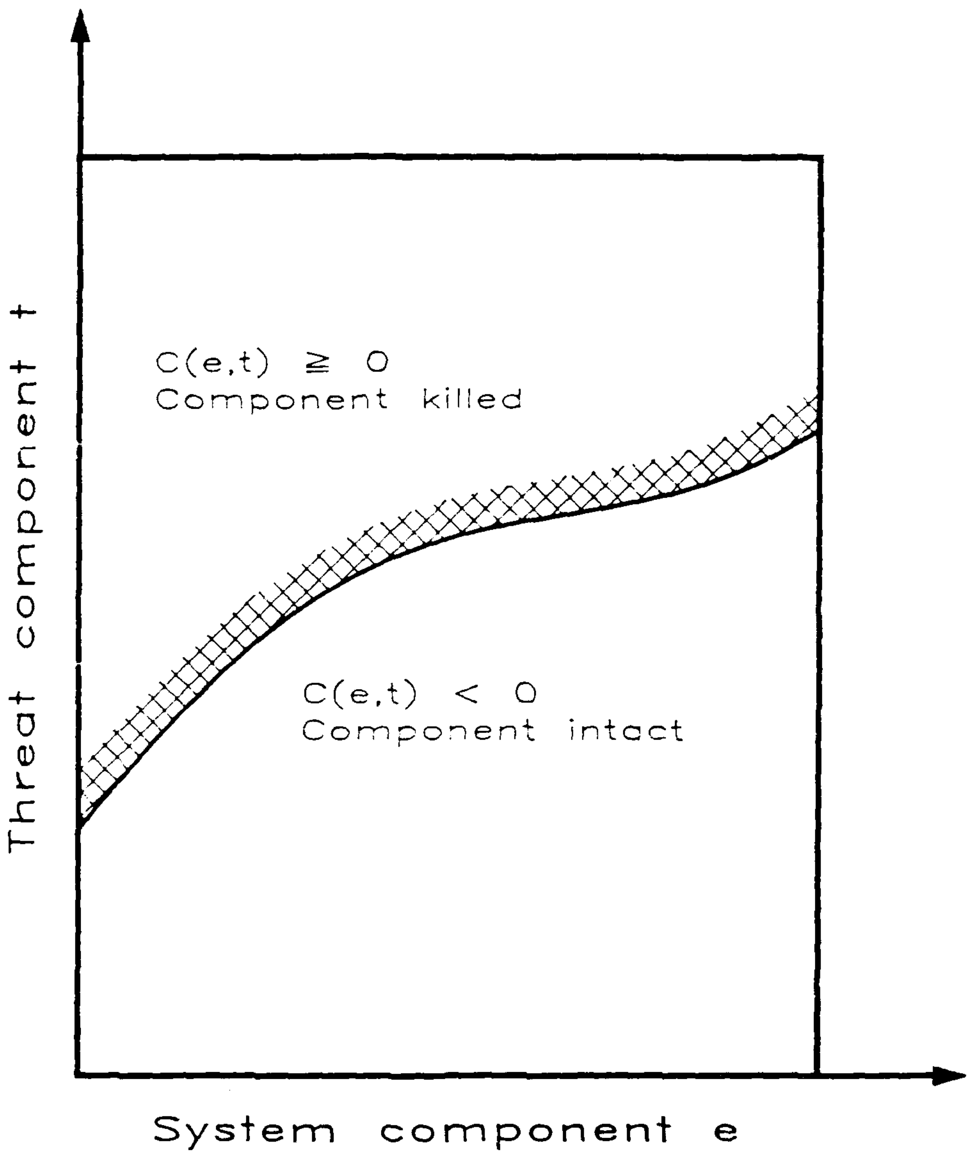

Figure 1. Decision Bound in a 2-D Event Space

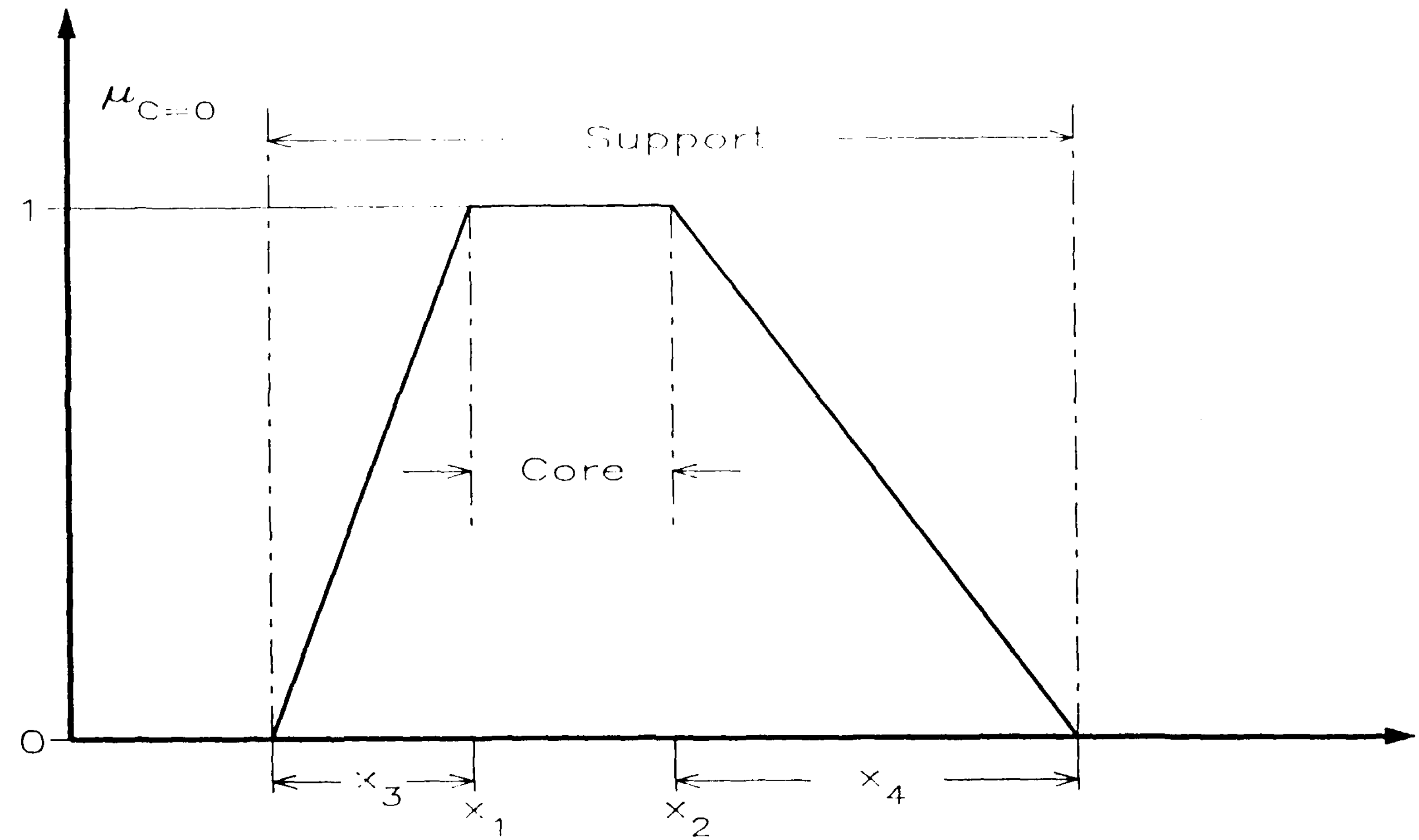

Figure 2. Trapezoidal Membership Function of a Decision Bound

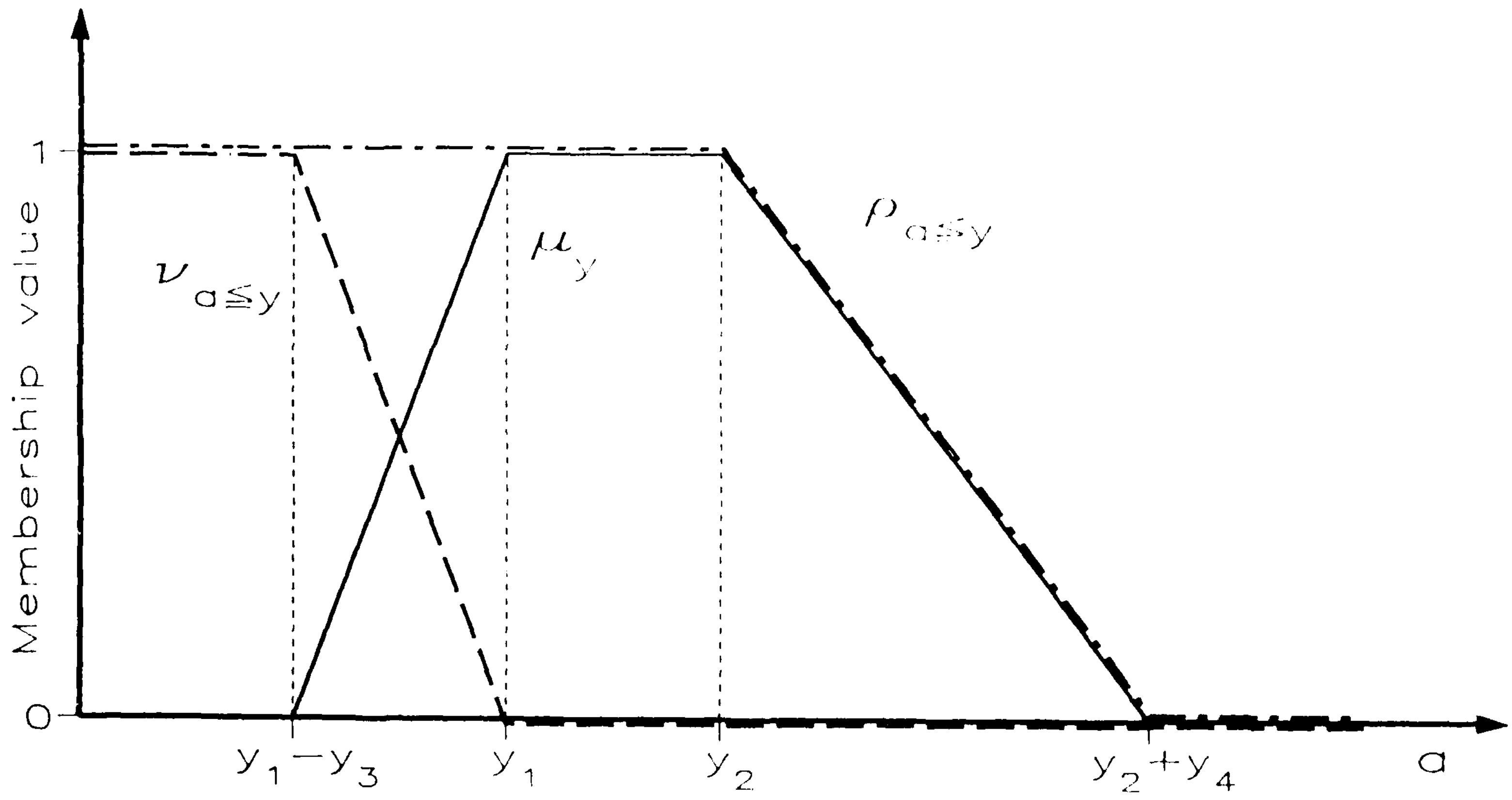

Figure 3. Possibility and Necessity of Crisp a ≤ Fuzzy y

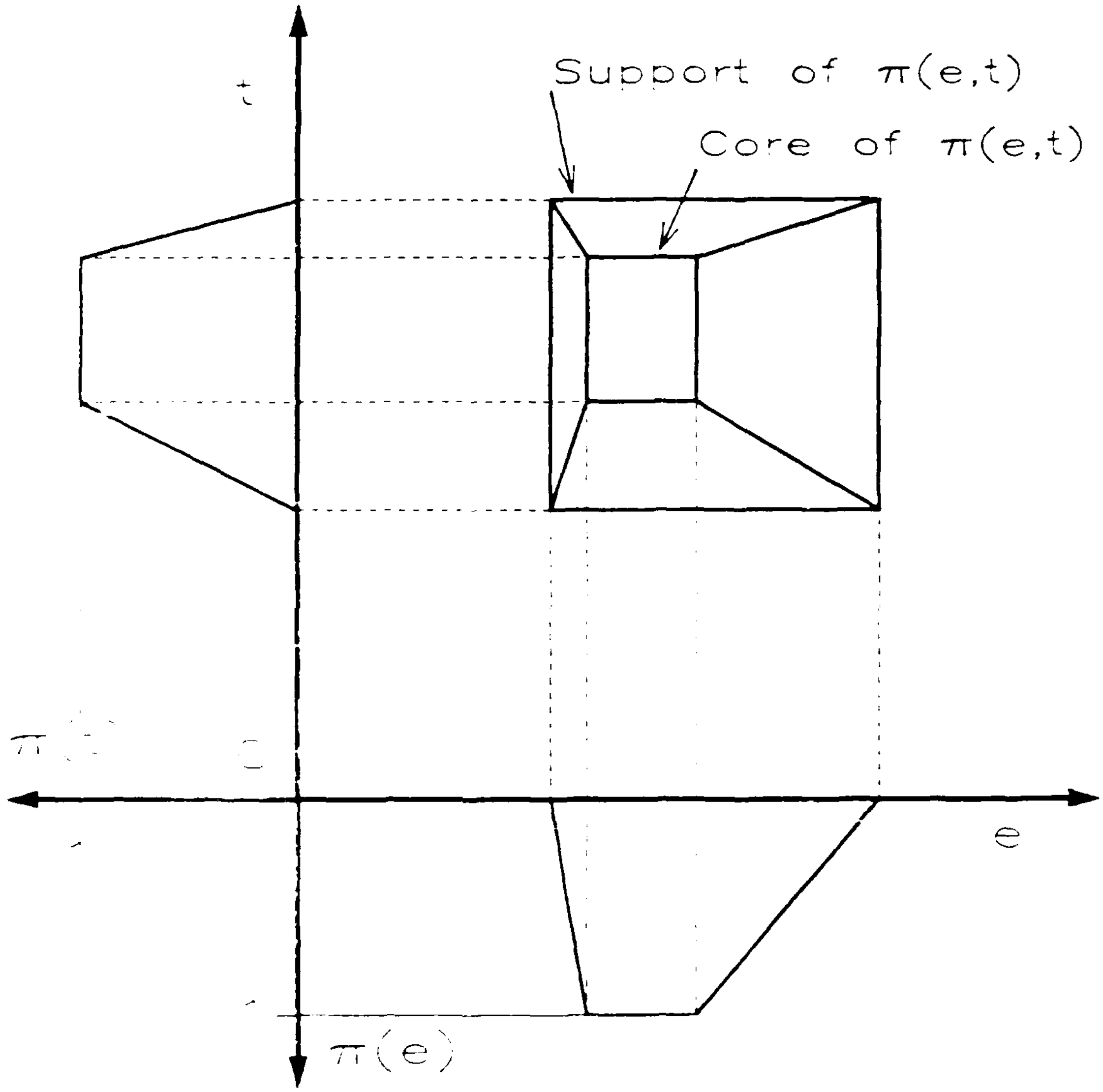

Figure 4. Possibility of a 2-D Event

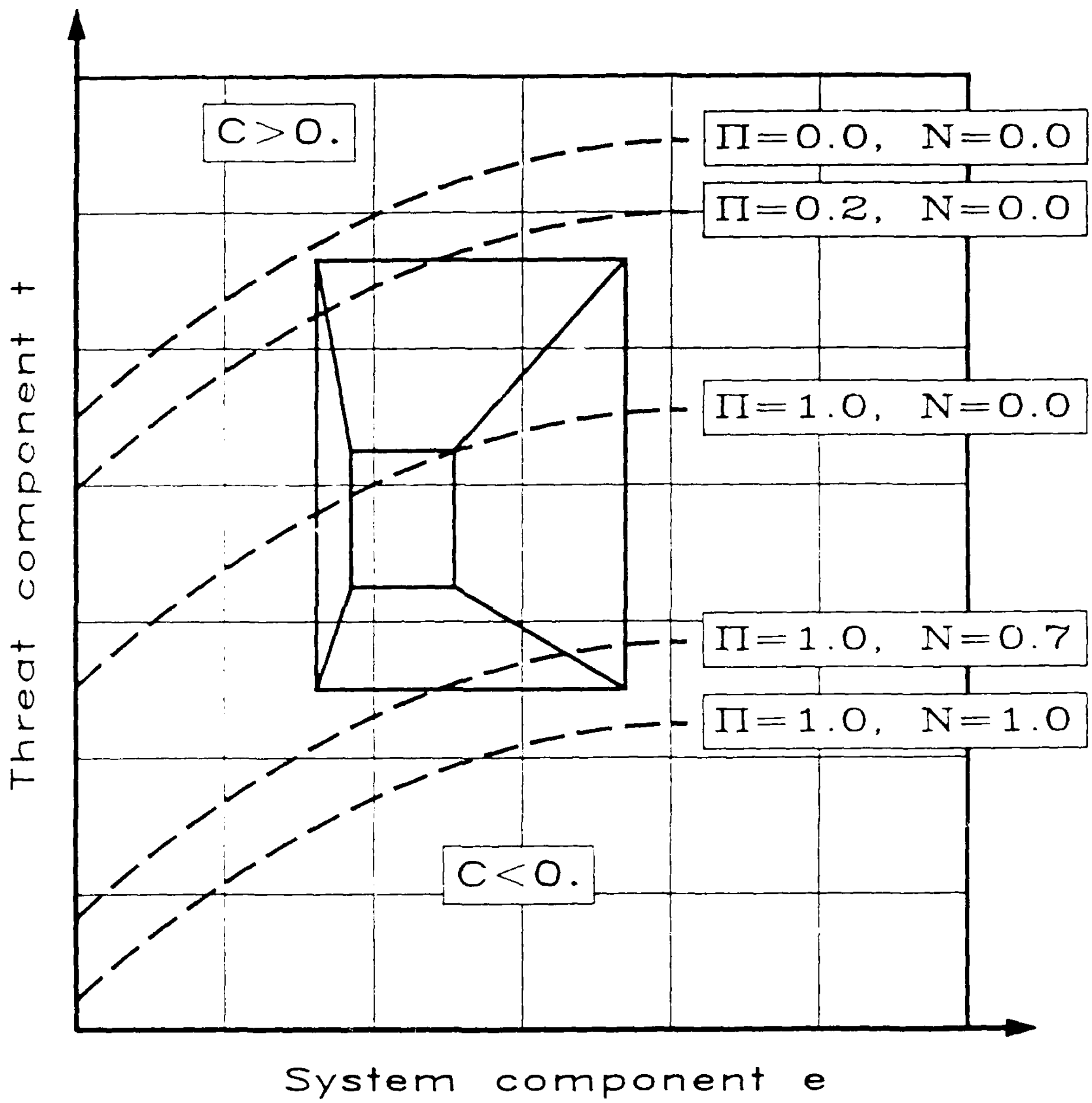

Figure 5. Possibility Measures for "Kill" in Case of a Fuzzy Event and Crisp Decision Bound.

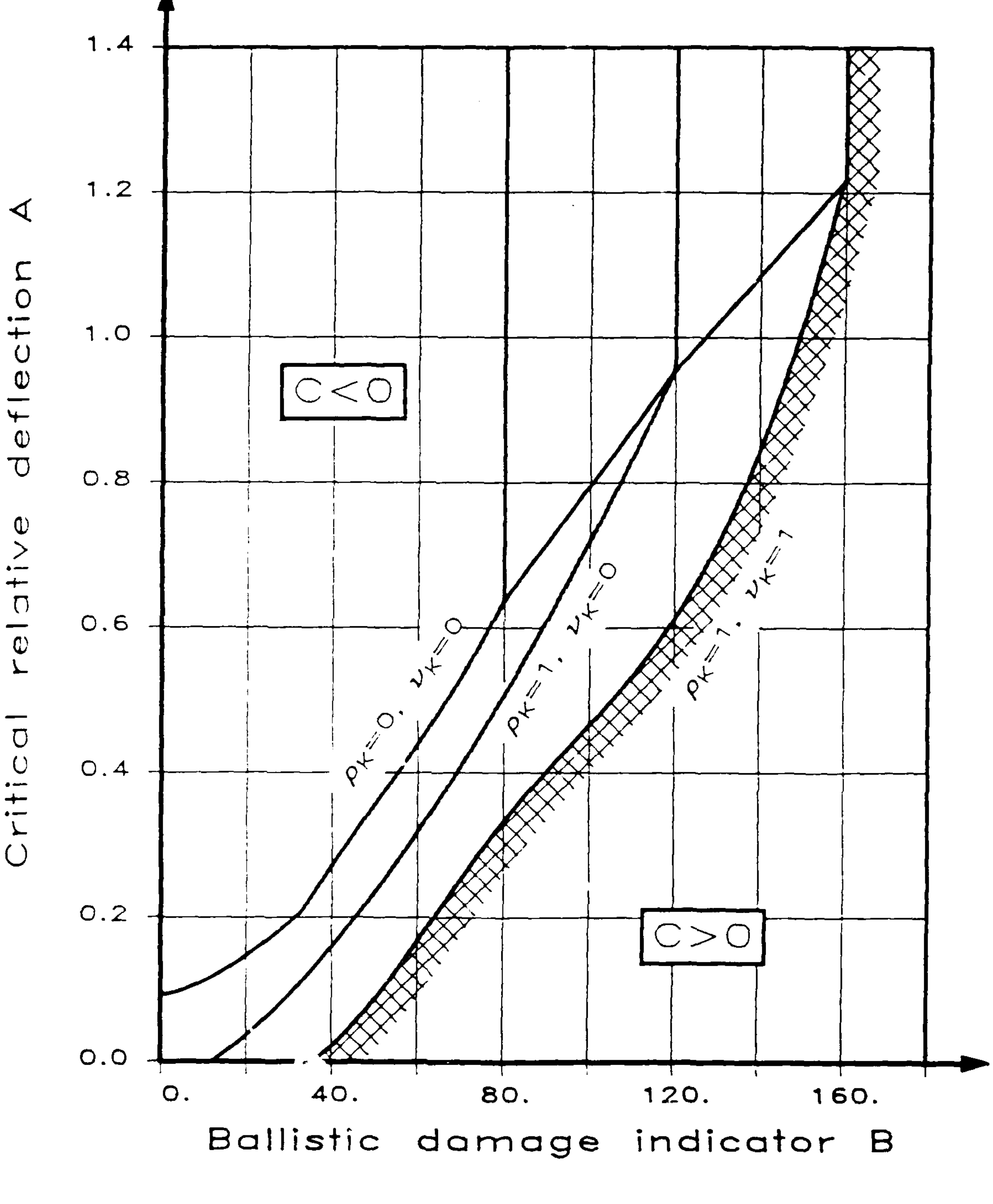

Figure 6. Example of a Fuzzy Decision Bound

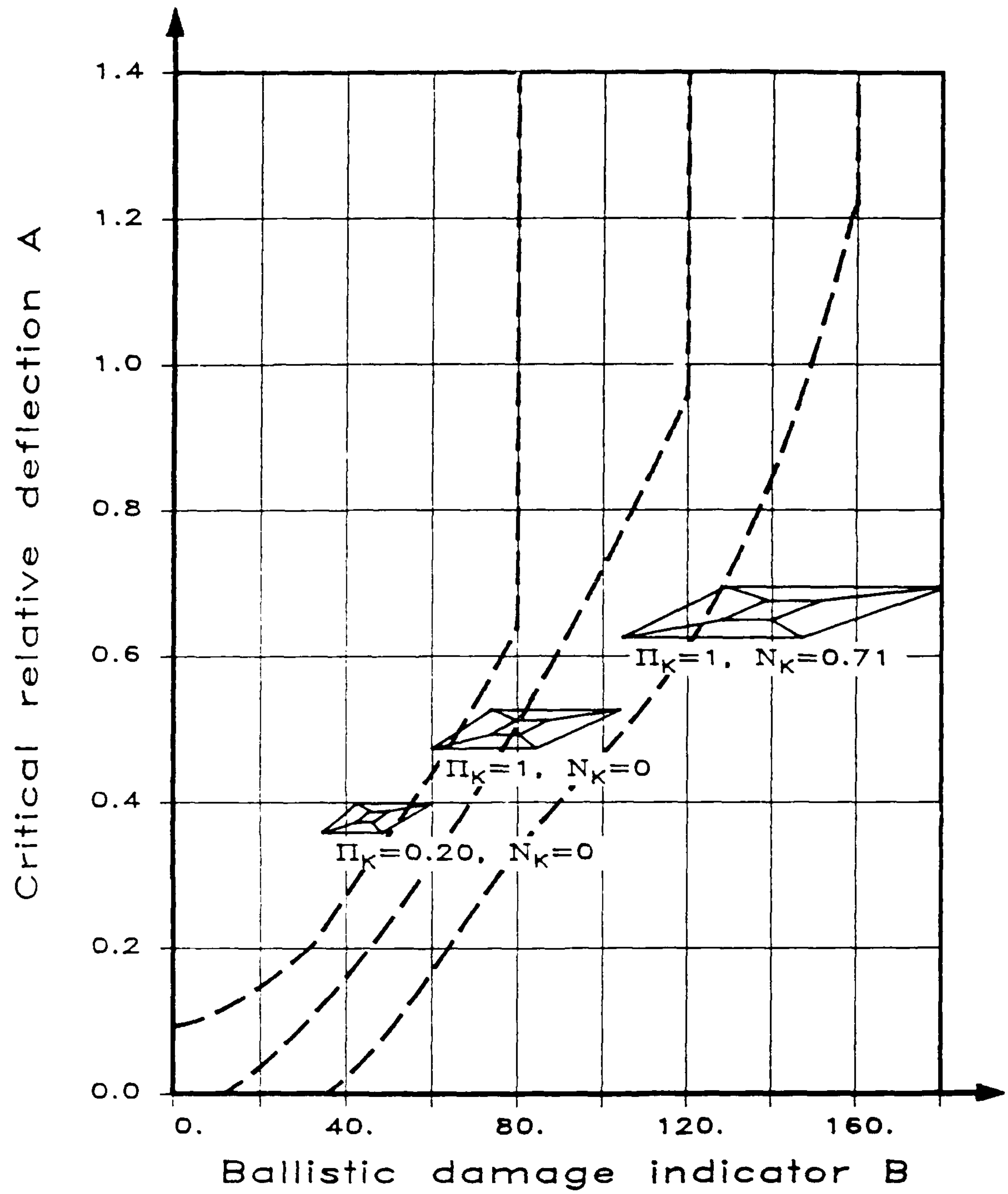

Figure 7. Fuzzy Event with Interactive Components

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